

# Mathematics Grade- and Course-Level Expectations

The *Mathematics Grade and Course Level Expectations* outline related ideas, concepts, skills and procedures that form the foundation for understanding and learning mathematics. They provide a framework to bring focus to teaching, learning, and assessing mathematics. The Grade Level Expectations (GLEs) in grades K-8 specify mathematical content that students need to understand deeply and thoroughly for future mathematics learning. The Course Level Expectations (CLEs) for Algebra I, Geometry, and Algebra II, as well as Integrated Math II and Integrated Math III **to be posted at a later date**, outline mathematics expectations for students enrolled in both traditional and integrated mathematics programs.

Since the Outstanding Schools Act of 1993, several documents have been developed prior to the 2004 K-12 *Grade Level Expectations* to aid Missouri school districts in creating curriculum that will enable all students to achieve their maximum potential. Those include:

- The *Show-Me Standards* which identify broad content knowledge and process skills for all students to be successful as they continue their education, enter the workforce, and assume civic responsibilities
- The *Framework for Curriculum Development* which provides districts with a “frame” for building curricula using the *Show-Me Standards* as a foundation
- The *Assessment Annotations for the Curriculum Frameworks* which identify content and processes that should be assessed at the local and state level in grades 4, 8, and 10 mathematics

Essential content, aligned to state and national documents that support inquiry-based instruction, included in the Grade and Course Level Expectations should **be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations**. Each Grade and Course Level Expectation is aligned to the Show-Me Content and Process Standards (1996). In addition, a Depth-of-Knowledge level has been assigned to each grade or course level expectation. The Depth of Knowledge identifies the highest level at which the expectation will be assessed, based upon the demand of the GLE. Depth-of-Knowledge levels include: Level 1-recall; Level 2-skill/concept; Level 3-strategic thinking; and Level 4-extended thinking.

Each GLE or CLE has been coded to identify those assessed at the state or local level. Those coded with an asterisk \*, indicate that it should be assessed at the local level. Those with no asterisk, indicate an expectation that will be assessed at the state level on a 3<sup>rd</sup> – 8<sup>th</sup> grade MAP Assessment or End-of-Course Exam. It is essential to include all expectations in your course or grade level curriculum, as they are important components in the understanding and learning of mathematics.

Sources: *College Board Standards for College Success: Mathematics and Statistics* (College Board, 2006). *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* (National Council of Teachers of Mathematics, 2007); *Indicators of College Readiness within Missouri's Two-Year Colleges* (Missouri Development Education Consortium); *Depth-of-Knowledge Levels* (Norman Webb); *Mathematics Engineering Technology & Science (METS) Alliance Report* (2006); *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000); *Show-Me Standards* (Missouri Department of Elementary and Secondary Education).

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# Number and Operations

1. Understand numbers, ways of representing numbers, relationships among numbers and number systems													
	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Algebra I	Geometry	Algebra II	
<b>A</b>	*rote count to 100 and recognize numbers up to 31	*read, write, and compare whole numbers less than 100	*read, write, and compare whole numbers less than 1000	*read, write and compare whole numbers up to 10,000	read, write and compare and whole numbers less than 100,000	*read, write and compare whole numbers less than 1,000,000, <u>unit fractions</u> and decimals to hundredths (including location on the number line)	apply and understand whole numbers to millions, fractions and decimals to the thousandths (including location on the number line)	compare and order all <u>positive rational numbers</u> and find their approximate location on a number line	*compare and order all rational numbers including percents, and find their approximate location on a number line	compare and order rational and irrational numbers, including finding their approximate locations on a number line	compare and order rational and irrational numbers, including finding their approximate locations on a number line	compare and order rational and irrational numbers, including finding their approximate locations on a number line	
Read, write and compare numbers													
DOK	1	1	1	1	1	1	1	1	1	1	1	1	
ST	MA 5 1.10	MA 5 1.10	MA 5 1.10	MA 5 1.10	MA 5 1.10	MA 5 1.10	MA 5 1.10	MA 5 1.10	MA 5 1.10	MA 5 1.10	MA 5 1.10	MA 5 1.10	
<b>B</b>	*recognize $\frac{1}{2}$ of a shape	*recognize $\frac{1}{2}$ and $\frac{1}{4}$ of a shape	*recognize unit fractions of a shape	*represents halves, thirds and fourths	*use models, benchmarks (0, $\frac{1}{2}$ and 1) and equivalent forms to judge the size of fractions	recognize and generate equivalent forms of <u>commonly used</u> fractions and decimals	recognize and generate equivalent forms of fractions, decimals and <u>benchmark</u> percents	recognize and generate equivalent forms of fractions, decimals and percents	*use fractions, decimals and percents to solve problems	use real numbers and various models, drawing, etc. to solve problems	use real numbers and various models, drawing, etc. to solve problems	use real numbers and various models, drawing, etc. to solve problems	
Represent and use rational numbers													
DOK	1	1	1	1	2	2	2	2	2	3	3	3	
ST	MA 5 1.10	MA 5	MA 5 1.10	MA 5 1.10	MA 5 1.10	MA 5 1.10	MA 5 1.10	MA 5 1.10	MA 5 3.3	MA 5 3.3	MA 5 3.3	MA 5 3.3	
<b>C</b>	*use <u>concrete objects</u> to <u>compose and decompose</u> values up to 10	* <u>compose</u> or <u>decompose</u> whole numbers up to 20 using multiple strategies such as known facts, doubles and <u>close to doubles</u> , tens, and one place value	* <u>compose</u> or <u>decompose</u> numbers by using a variety of strategies, such as using known facts, tens place value or <u>landmark numbers</u> to solve problems	recognize equivalent representations for the same number and generate them by <u>decomposing and composing numbers</u> including expanded notation	recognize equivalent representations for the same number and generate them by <u>decomposing and composing numbers</u>	*recognize equivalent representations for the same number and generate them by <u>decomposing and composing numbers</u>	*recognize equivalent representations for the same number and generate them by <u>decomposing and composing numbers</u>	*recognize equivalent representations for the same number and generate them by <u>decomposing and composing numbers</u> , including exponential notation	*recognize equivalent representations for the same number and generate them by <u>decomposing and composing numbers</u> , including scientific notation	*use a variety of representations to demonstrate an understanding of very large and very small numbers		*use a variety of representations to demonstrate an understanding of very large and very small numbers	
Compose and decompose numbers													
DOK	2	2	2	2	2	2	2	2	2	2		2	
ST	MA 1 1.6	MA 1 1.6	MA 1 1.6	MA 1 1.6	MA 1 1.6	MA 5 1.6	MA 5 1.6	MA 5 1.6	MA 5 1.6	MA 5 1.6		MA 5 1.6	

# Number and Operations

1. Understand numbers, ways of representing numbers, relationships among numbers and number systems -- continued												
	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Algebra I	Geometry	Algebra II
<b>D</b>		*skip count by 2s, 5s and 10s	*skip count by multiples of numbers less than 10	* <u>classify numbers</u> by their characteristics, including odd and even	*classify and describe numbers by their characteristics, including <u>odd, even, multiples and factors</u>	*describe numbers according to their characteristics, including whole number <u>common factors and multiples, prime or composite, and square numbers</u>						
Classify and describe numeric relationships												
DOK		1	1	1	2	2						
ST		MA 5 1.6	MA 5 1.6	MA 5 1.6	MA 5 1.10	MA 5 1.10						

# Number and Operations

2. Understand meanings of operations and how they relate to one another													
	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Algebra I	Geometry	Algebra II	
A		*represent/ model a given situation involving addition and subtraction of whole numbers using pictures, objects, or symbols	*represent/ model a given situation involving two- digit whole number addition or subtraction	*represent/ model a given situation in- volving multi- plication and related division using various models includ- ing sets, arrays, areas, repeated addition/sub- traction, sharing and partitioning	*represent and recognize multiplication and related division using various models, including equal intervals on the number line, equal size groups, distributive property, etc.	represent and recognize division using various models, including <u>quotative</u> and <u>partitive</u>							
DOK		2	2	2	2	2							
ST		MA 1 1.10	MA 1 1.10	MA 1 1.0	MA 1 1.10	MA 1 1.10							
B				*describe the effects of adding and subtracting whole numbers as well as the relationship between the two operations	describe the effects of multiplying and dividing whole numbers as well as the relationship between the two operations	*describe the effects of addition and subtraction on fractions and decimals	describe the effects of multiplication and division on fractions and decimals	*describe the effects of all operations on <u>rational numbers</u> including integers		*describe the effects of operations, such as multiplication, division, and computing powers and roots on the magnitude of quantities			
DOK		2		2	2	2	2	2		2			
ST		MA 1 1.10		MA 1 1.10	MA 1 1.10	MA 1 1.10	MA 1 1.10	MA 1 1.10		MA 1 1.10			
C							*apply <u>properties of operations</u> (including order of operations) to positive rational numbers	apply <u>properties of operations</u> (including order of operations) to positive rational numbers and integers	apply <u>properties of operations</u> to all rational numbers including order of operations and inverse operations				
DOK							2	2	2				
ST							MA 1 1.10	MA 1 1.10	MA 1 1.10				

Number and Operations

2. Understand meanings of operations and how they relate to one another -- continued													
	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Algebra I	Geometry	Algebra II	
D							identify square and cubic numbers and determine whole number roots and cubes	*approximate the value of square roots to the nearest whole number		*apply operations to real numbers, using mental computation or paper-and-pencil calculations for simple cases and technology for more complicated cases	*apply operations to real numbers, using mental computation or paper-and-pencil calculations for simple cases and technology for more complicated cases	*apply operations to matrices and complex numbers, using mental computation or paper-and-pencil calculations for simple cases and technology for more complicated cases	
Apply operations on real and complex numbers													
DOK							1	1		2	2	2	
ST							M 5 1.6	MA 5 1.6		MA 1 1.10	MA 1 1.10	MA 1 1.10	

# Number and Operations

3. Compute fluently and make reasonable estimates													
	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Algebra I	Geometry	Algebra II	
A		*describe or represent the mental strategy used to compute addition and subtraction problems	*describe or notate the mental strategy used to compute addition or subtraction of whole numbers, including 2-digit numbers	*represent a mental strategy used to compute a given multiplication problem up to 9 x 9	*represent a mental strategy used to compute a given multiplication problem (up to 2-digit by 2-digit multiple of)	*describe a mental strategy used to compute a given division problem, where the quotient is a multiple of 10 and the divisor is a 1-digit number (e.g., 350 /7)							
DOK		2	2	2	2	2							
ST		MA 1 3.2	MA 1 3.2	MA 1 3.2	MA 1 3.2	MA 1 3.2							
B	*connect number words (orally) and quantities they represent	*use strategies to develop fluency with basic number relationships of addition and subtraction for sums up to 20	*demonstrate fluency including quick recall with basic number relationships of addition and subtraction for sums up to 20	use strategies develop fluency with basic number relationships (9 X 9) of multiplication and division	demonstrate fluency with basic number relationships (12 X 12) of multiplication and related division facts	demonstrate fluency with efficient procedures for adding and subtracting decimals and fractions (with unlike denominators) and division of whole numbers							
DOK	1	1	1	1	1	1							
ST	MA 1 1.10	MA.1 1.6	MA.1 1.6	MA.1 1.6	MA.1 1.6	MA 1 1.6							
C		*apply and describe the strategy used to solve addition or subtraction problems	*apply and describe the strategy used to compute 3 2-digit addition or subtraction problems with regrouping	apply and describe the strategy used to compute up to 3-digit addition or subtraction problems	apply and describe the strategy used to compute a given multiplication of 2-digit by 2-digit numbers and related division facts	apply and describe the strategy used to compute a division problem up to a 3- digit by 2-digit and addition and subtraction of fractions and decimals	multiply and divide positive rational numbers	apply all operations on rational numbers including integers					
DOK		2	2	2	2	2	1	2					
ST		MA 1 3.2	MA 1 3.2	MA 1 3.2	MA 1 3.2	MA 1 3.2	MA 1 3.1	MA 1 3.1					

# Number and Operations

3. Compute fluently and make reasonable estimates -- continued												
	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Algebra I	Geometry	Algebra II
<b>D</b>			*estimate sums and differences of whole numbers	estimate and justify sums and differences of whole numbers	estimate and justify products of whole numbers	estimate and justify products, and quotients of whole numbers and sums differences of decimals and fractions	*estimate and justify the results of multiplication and division of positive rational numbers	*estimate and justify the results of all operations on rational numbers		*judge the reasonableness of numerical computations and their results	*judge the reasonableness of numerical computations and their results	*judge the reasonableness of numerical computations and their results, including complex numbers
Estimate and justify solutions												
DOK			3	3	3	3	3	3		3	3	3
ST			MA 1 3.2	MA 1 3.2	MA 1 3.2	MA 1 3.2	MA 1 3.2	MA 1 3.2		MA 1 3.2	MA 1 3.2	MA 1 3.2
<b>F</b>							solve problems using ratios and rates	solve problems involving proportions, such as scaling and finding equivalent ratios		*solve problems involving proportions	*solve problems involving proportions	*solve problems involving proportions
Use proportional reasoning												
DOK							2	2		2	2	2
ST							MA 1 3.2	MA 1 3.2		MA 1 3.2	MA 1 3.2	MA 1 3.2

# Algebraic Relationships

1. Understand patterns, relations and functions													
	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Algebra I	Geometry	Algebra II	
<b>A</b>	*recognize or repeat sequences of sounds or shapes	*extend patterns of sound, shape, motion or a simple numeric pattern	*describe and extend simple numeric patterns and change from one representation to another	*extend geometric (shapes) and numeric patterns to find the next term	*describe geometric and numeric patterns	make and describe <u>generalizations</u> about geometric and numeric patterns							
Recognize and extend patterns													
DOK	2	2	2	2	2	2							
ST	MA 4 1.6	MA 4 1.6	MA 4 1.6	MA 4 1.6	MA 4 1.6	MA 4 1.6							
<b>B</b>	*create and continue patterns	*describe how simple <u>repeating patterns</u> are generated	*describe how simple <u>growing patterns</u> are generated	*represent patterns using words, tables or graphs	*analyze patterns using words, tables and graphs	represent and analyze patterns using words, tables and graphs	represent and describe patterns with tables, graphs, pictures, <u>symbolic rules</u> or words	analyze patterns represented <u>graphically</u> or <u>numerically</u> with words or <u>symbolic rules</u> , including <u>recursive notation</u>	generalize patterns represented <u>graphically</u> or <u>numerically</u> with words or <u>symbolic rules</u> , using <u>explicit notation</u>	generalize patterns using <u>explicitly</u> or <u>recursively</u> defined functions	generalize patterns using <u>explicitly</u> or <u>recursively</u> defined functions	generalize patterns using <u>explicitly</u> or <u>recursively</u> defined functions	
Create and analyze patterns													
DOK	2	2	2	2	3	3	2	3	2	2	2	2	
ST	MA 4 1.6	MA 4 1.6	MA 4 1.6	MA 4 1.6	MA 4 1.6	MA 4 1.6	MA 4 1.6	MA 4 1.6	MA 4 1.6	MA 4 1.6	MA 4 1.6	MA 4 1.6	
<b>C</b>							*compare various forms of <u>representations</u> to identify patterns	compare and contrast various forms of <u>representations</u> of patterns	compare and contrast various forms of <u>representations</u> of patterns	compare and contrast various forms of <u>representations</u> of patterns	compare and contrast various forms of <u>representations</u> of patterns	compare and contrast various forms of <u>representations</u> of patterns	
Classify objects and representations													
DOK							2	3	3	3	3	3	
ST							MA 4 1.6	MA 4 1.6	MA 4 1.6	MA 4 1.6	MA 4 1.6	MA 4 1.6	



Algebraic Relationships

1. Understand patterns, relations and functions -- continued													
	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Algebra I	Geometry	Algebra II	
D							*identify <u>functions</u> as <u>linear</u> or <u>nonlinear</u> from tables or graphs	identify <u>functions</u> as <u>linear</u> or <u>nonlinear</u> from tables, graphs or equations	identify <u>functions</u> as <u>linear</u> or <u>nonlinear</u> from tables, graphs or equations	understand and compare the properties of <u>linear</u> and <u>nonlinear functions</u>	apply appropriate <u>properties of exponents</u> to simplify expressions and solve equations	compare properties of linear, exponential, logarithmic and rational functions	
Identify and compare functions													
DOK							1	1	1	2	2	2	
ST							MA 4 1.6	MA 4 1.6	MA 4 1.6	MA 4 1.6	MA 4 1.6	MA 4 1.6	
E										describe the effects of <u>parameter changes</u> on <u>linear</u> , <u>exponential growth/decay</u> and <u>quadratic</u> functions including intercepts		describe the effects of <u>parameter changes</u> on functions	
Describe the effects of parameter changes													
DOK										2		2	
ST										MA 4 1.6		MA 4 1.6	

## Algebraic Relationships

2. Represent and analyze mathematical situations and structures using algebraic symbols													
	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Algebra I	Geometry	Algebra II	
<b>A</b>		*using addition or subtraction, represent a mathematical situation as an <u>expression</u> or number sentence	*using addition or subtraction, represent a mathematical situation as an <u>expression</u> or number sentence	using all operations, represent a mathematical situation as an <u>expression</u> or number sentence	using all operations, represent a mathematical situation as an <u>expression</u> or number sentence	using all operations, represent a mathematical situation as an <u>expression</u> or number sentence using a letter or symbol	use <u>symbolic algebra</u> to represent unknown quantities in expressions or equations and solve one-step equations	use <u>symbolic algebra</u> to represent unknown quantities in expressions or equations and solve linear equations with one variable	use <u>symbolic algebra</u> to represent and solve problems that involve linear relationships	use <u>symbolic algebra</u> to represent and solve problems that involve linear and quadratic relationships including equations and inequalities		use <u>symbolic algebra</u> to represent and solve problems that involve exponential, quadratic and logarithmic relationships	
DOK		2	2	2	2	2	2	2	3	3		3	
ST		MA 4 1.10	MA 4 1.10	MA 4 1.10	MA 4 1.10	MA 4 1.10	MA 4 3.3	MA 4 3.3	MA 4 3.3	MA 4 3.3		MA 4 3.3	
<b>B</b>		*apply the commutative and associative properties of addition to whole numbers	*solve problems with whole numbers using the commutative and associative properties of addition	* use the <u>commutative, distributive and associative</u> properties for basic facts of whole numbers	*use the <u>commutative, distributive and associative</u> properties of addition and multiplication for multidigit numbers	*use the <u>commutative, distributive and associative</u> properties for fractions and decimals	use the <u>commutative, distributive and associative</u> properties to generate equivalent forms for simple algebraic expressions	use properties to generate equivalent forms for simple algebraic expressions that include positive rationals and integers	use properties to generate equivalent forms for simple algebraic expressions that include all rationals	describe and use algebraic manipulations, including factoring and rules of integer exponents and apply <u>properties of exponents</u> (including order of operations) to simplify expressions		describe and use algebraic manipulations, <u>inverse</u> or <u>composition</u> of functions	
DOK		2	2	2	2	2	2	2	2	2		2	
ST		MA 4 1.10	MA 4 3.2	MA 4 3.2	MA 4 3.2	MA 4 3.2	MA 4 3.2	MA 4 3.2	MA 4 3.2	MA 4 3.2		MA 4 3.2	

Algebraic Relationships

2. Represent and analyze mathematical situations and structures using algebraic symbols -- continued													
	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Algebra I	Geometry	Algebra II	
C										use and solve equivalent forms of equations (linear, absolute value, and quadratic)		use and solve equivalent forms of equations and inequalities	
Utilize equivalent forms													
DOK										2		2	
ST										MA 4 3.2		MA 4 3.2	
D										use and solve systems of linear equations or inequalities with 2 variables		use and solve systems of linear and quadratic equations or inequalities with 2 variables	
Utilize systems													
DOK										2		3	
ST										MA 4 1.6		MA 4 1.6	

# Algebraic Relationships

3. Use mathematical models to represent and understand quantitative relationships													
	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Algebra I	Geometry	Algebra II	
A	* <u>model</u> situations that involve whole numbers, using pictures, objects or symbols	* <u>model</u> situations that involve the addition of whole numbers, using pictures, objects or symbols	* <u>model</u> situations that involve addition and subtraction of whole numbers, using pictures, objects or symbols	* <u>model</u> problem situations, including multiplication with objects or drawings	* <u>model</u> problem situations, using representations such as graphs, tables or number sentences	<u>model</u> problem situations and draw conclusions, using representations such as graphs, tables or number sentence	<u>model</u> and solve problems, using multiple representations such as tables, expressions and one-step equations	<u>model</u> and solve problems, using multiple representations such as graphs, tables, expressions, and linear equations	<u>model</u> and solve problems, using multiple representations such as graphs, tables, and linear equations	identify quantitative relationships and determine the type(s) of functions that might model the situation to solve the problem	identify quantitative relationships and determine the type(s) of functions that might model the situation to solve the problem	identify quantitative relationships and determine the type(s) of functions that might model the situation to solve the problem	
DOK	2	2	2	2	2	3	2	2	2	2	2	2	
ST	MA 1 1.6	MA 1 1..6	MA 1 1.6	MA 4 1.6	MA 4 1.6	MA 4 1.6	MA 4 3.6	MA 4 1.6,3.6	MA 4 3.6	MA 4 1.6	MA 4 1.6	MA 4 1.6	

4. Analyze change in various contexts													
	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Algebra I	Geometry	Algebra II	
A			*describe <u>qualitative</u> change, such as students growing taller	*describe <u>quantitative</u> change, such as students growing two inches in a year	*describe mathematical relationships in terms of constant rates of change	*identify, model and describe situations with constant or varying rates of change	*construct and analyze representations to compare situations with constant or varying rates of change	compare situations with constant or varying rates of change	analyze the nature of changes (including slope and intercepts) in quantities in linear relationships	analyze linear and quadratic functions by investigating rates of change, intercepts and zeros	analyze linear functions by investigating rates of change and intercepts	analyze exponential and logarithmic functions by investigating rates of change, intercepts and asymptotes	
DOK			2	2	2	3	3	3	3	3	3	3	
ST			MA 4 1.6	MA 4 1.6	MA 4 1.6	MA 4 1.6	MA 4 1.6	MA 4 1.6	MA 4 1.6	MA 4 1.6	MA 4 1.6	MA 4 1.6	

# Geometric and Spatial Relationships

1. Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships												
	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Algebra I	Geometry	Algebra II
<b>A</b>	*identify and describe 2- and 3-dimensional shapes using physical models (circle, rhombus, rectangle, triangle, sphere, rectangular prism, cylinder, pyramid) that represent shapes in their environment	*identify, name and describe 2- and 3-dimensional shapes using physical models (circle, triangle, trapezoid, rectangle, rhombus, sphere, rectangular prism, cylinder, pyramid)	*describe <u>attributes</u> and <u>parts</u> of 2- and 3-dimensional shapes (circle, triangle, trapezoid, rectangle, rhombus, sphere, rectangular prism, cylinder, pyramid)	compare and analyze 2-dimensional shapes by describing their <u>attributes</u> (circle, rectangle, rhombus, trapezoid, triangle)	name and identify <u>properties of 1-, 2- and 3-dimensional shapes</u> and describe the <u>attributes</u> of 2- and 3-dimensional shapes using appropriate geometric vocabulary (rectangular prism, cylinder, pyramid, sphere, cone, parallelism, perpendicularity)	*analyze and classify 2- and 3-dimensional shapes by describing the <u>attributes</u>	*identify similar and congruent shapes	*identify the 2-dimensional cross-section of a 3-dimensional shape	*describe, classify and generalize relationships between and among types of a) 2-dimensional objects and b) 3-dimensional objects using their defining <u>properties</u> including Pythagorean Theorem		use inductive and deductive reasoning to establish the validity of geometric <u>conjectures</u> , prove theorems and critique arguments made by others	use trigonometric relationships with right triangles to determine lengths and angle measures
DOK	2	2	2	2	2	2	1	2	3		3	2
ST	MA 2 1.10	MA 2 1.10	MA 2 1.10	MA 2 1.10	MA 2 1.10	MA 2 1.10	MA 2 1.10	MA 2 1.10	MA 2 1.6		MA 2 3.5	MA 2 3.2
<b>B</b>								describe relationships between <u>corresponding sides</u> , <u>corresponding angles</u> and corresponding perimeters of <u>similar polygons</u>		*apply geometric properties such as similarity and angle relationship to solve multi-step problems in 2 dimensions		
DOK								2		2		
ST								MA 2 1.6		MA 2 3.6		

# Geometric and Spatial Relationships

1. Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships – continued													
	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Algebra I	Geometry	Algebra II	
C		*use models to compose and decompose 2-dimensional shapes		*predict the results of putting together or taking apart 2- and 3-dimensional shapes	*describe the results of subdividing, combining and <u>transforming shapes</u>	predict and justify the results of subdividing, combining and <u>transforming shapes</u>							
DOK		2		3	2	3							
ST		MA 2 1.6		MA 2 1.6	MA 2 1.6	MA 2 1.6							

2. Specify locations and describe spatial relationships using coordinate geometry and other representational systems													
	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Algebra I	Geometry	Algebra II	
A	*describe, name and interpret relative positions in space (above, below, front, behind)	*describe, name and interpret relative positions in space (left, right)	*identify locations with simple relationships on a map (coordinate system)	*describe location using common language and geometric vocabulary (forward, back, left, right, north, south, east, west)	*describe movement using common language and geometric vocabulary (forward, back, left, right, north, south, east, west)	*use <u>coordinate systems</u> to specify locations, describe paths and find the distance between points along horizontal and vertical lines	*use <u>coordinate systems</u> to construct geometric shapes	use coordinate geometry to construct and identify geometric shapes in the <u>coordinate plane</u> using their properties	use coordinate geometry to analyze <u>properties of right triangles</u> and quadrilaterals (including the use of the Pythagorean Theorem)		make conjectures and solve problems involving 2-dimensional objects represented with Cartesian coordinates		
DOK	2	2	1	2	2	2	2	2	2		3		
ST	MA 2 1.10	MA 2 1.10	MA 2 3.1	MA 2 1.10	MA 2 3.3	MA 2 1.10	MA 2 1.10	MA 2 3.2	MA 2 3.2		MA 2 3.3		

## Geometric and Spatial Relationships

3. Apply transformations and use symmetry to analyze mathematical situations													
	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Algebra I	Geometry	Algebra II	
<b>A</b>	*use manipulatives to recognize from different perspectives and orientations models of slides and turns	*use manipulatives to model flips	*use manipulatives to model slides and turns	*determine if two objects are <u>congruent</u> through a slide, flip or turn	predict the results of <u>sliding/ translating/ flipping/ reflecting or turning/ rotating around the center point</u> of a polygon	*predict, draw and describe the results of <u>sliding/ translating/ flipping/ reflecting and turning/ rotating around a center point</u> of a polygon	*describe the transformation from a given pre-image using the terms <u>reflection/flip, rotation/turn, and translation/ slide</u>		reposition shapes under <u>formal</u> transformations such as reflection, rotation and translation		use and apply constructions and the coordinate plane to represent translations, reflections, rotations and dilations of objects		
DOK	2	2	2	2	3	3	3		2		2		
ST	MA 2 1.6	MA 2 1.6	MA 2 1.6	MA 2 3.2	MA 2 3.6	MA 2 3.6	MA 2 3.3		MA 2 3.3		MA 2 1.10		
<b>B</b>								describe the relationship between the scale factor and the perimeter of the image using a <u>dilation (contractions-magnifications)</u> (stretching/ shrinking)	describe the relationship between the scale factor and the area of the image using a <u>dilation</u> (stretching/ shrinking)			translate, dilate and reflect <u>functions</u>	
DOK								2				2	
ST								MA 2 3.6				MA 4 3.1	
<b>C</b>		*recognize shapes that have symmetry	*create shapes that have symmetry	*identify lines of symmetry in polygons	create a figure with multiple lines of symmetry and identify the lines of symmetry	identify polygons and designs with <u>rotational symmetry</u>	*create polygons and designs with <u>rotational symmetry</u>	*determine all lines of symmetry of a polygons	*identify the number of rotational symmetries of regular polygons		identify types of symmetries of 2- and 3-dimensional figures		
DOK		1	2	1	2	1	2	1	1		2		
ST		MA 2 1.10	MA 2 1.10	MA 2 1.10	MA 2 1.10	MA 2 1.6	MA 2 1.6	MA 2 1.6	MA 2 1.6		MA 2 1.10		

## Geometric and Spatial Relationships

4. Use visualization, spatial reasoning and geometric modeling to solve problems													
	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Algebra I	Geometry	Algebra II	
<b>A</b>					*given the picture of a <u>prism</u> , identify the shapes of the faces	given a <u>net of a prism</u> or cylinder, identify the 3-dimensional shape	*use spatial visualization to identify <u>isometric representations</u> of <u>mat plans</u>	*use spatial visualizations to identify various 2-dimensional views of <u>isometric drawings</u>	create <u>isometric drawings</u> from a given <u>mat plan</u>		draw and use vertex-edge graphs or networks to find optimal solutions and draw representations of 3-dimensional geometric objects from different perspectives		
Recognize and draw three-dimensional representations													
DOK					1	2	2	2	3		3		
ST					MA 2 3.3	MA 2 3.3	MA 2 3.3	MA 2 3.3	MA 2 3.3		MA 2 4.1		
<b>B</b>							draw or use visual models to represent and solve problems	draw or use visual models to represent and solve problems	draw or use visual models to represent and solve problems	*draw or use visual models to represent and solve problems	*draw or use visual models to represent and solve problems	*draw or use visual models to represent and solve problems	
Draw and use visual models													
DOK							3	3	3	3	3	3	
ST							MA 2 3.3	MA 2 3.3	MA 2 3.3	MA 2 3.3	MA 2 3.3	MA 2 3.3	



# Measurement

1. Understand measurable attributes of objects and the units, systems and processes of measurement													
	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Algebra I	Geometry	Algebra II	
<b>A</b>	*compare and order objects according to their size or weight	*select the appropriate tool for the <u>attribute</u> being measured (size, temperature, time, weight)	*select an appropriate unit and tool for the <u>attribute</u> being measured (size, temperature, time, weight) and to the nearest inch, centimeter, degree, hour and pound	*identify, justify and use the appropriate unit of measure (linear, time, weight)	*identify and justify the unit of linear measure including perimeter and (customary metric)	*identify and justify the unit of measure for area (customary and metric)	identify and justify the unit of measure for area and volume (customary and metric)	*identify and justify the unit of measure for volume (customary and metric)					
Determine unit of measurement													
DOK	2	2	2	3	3	3	3	3					
ST	MA 2 1.8	MA 2 3.1	MA 2 3.1	MA 2 3.1	MA 2 3.1	MA 2 3.1	MA 2 3.1	MA 2 3.1					
<b>B</b>					*identify equivalent linear measures within a system of measurement	*identify the equivalent weights and equivalent capacities within a system of measurement		identify the equivalent area and volume measures within a system of measurement (e.g., sq ft. to sq in, m <sup>3</sup> to c m <sup>3</sup> )					
Identify equivalent measures													
DOK					1	1		1					
ST					MA 2 1.6	MA 2 1.6		MA 2 1.6					
<b>C</b>	*describe passage of time using terms such as today, yesterday, tomorrow	*tell time to the nearest half hour	*tell time to the nearest one fourth (quarter) hour	tell time to the nearest five minutes	tell time to the nearest minute		*solve problems involving elapsed time (hours and minutes)	*solve problems involving addition and subtraction of time (hours, minutes and seconds)					
Tell and use units of time													
DOK	2	1	1	1	1		2	2					
ST	MA 2 3.1	MA 2 1.10	MA 2 1.10	MA 2 1.10	MA 2 1.10		MA 5 3.1	MA 5 3.1					

Measurement

1. Understand measurable attributes of objects and the units, systems and processes of measurement -- continued													
	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Algebra I	Geometry	Algebra II	
D	*identify and know the value of a penny, nickel, dime, and quarter	*count money to a dollar, including half dollars	*make change from a dollar	determine change from \$5.00 and add and subtract money values to \$5.00	determine change from \$10.00 and add and subtract money values to \$10.00								
Count and compute money													
DOK	2	2	2	2	2								
ST	MA 1 1.10	MA 1 1.10	MA 1 1.10	MA 1 1.10	MA 1 1.10								

Measurement

2. Apply appropriate techniques, tools and formulas to determine measurements													
	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Algebra I	Geometry	Algebra II	
<b>A</b>	*measure objects by comparison of lengths (shorter, same, longer)  Use standard or non-standard measurement	*use repetition of a single unit to measure something larger than the unit, (e.g. length of book with paper clips)	*use standard units of measure (cm, inch) and the inverse relationships between the size and number of units	*use a <u>referent</u> for measures to make comparisons and estimates	*select and use <u>benchmarks</u> to estimate measurements (linear, capacity, weight)								
DOK	1	1	2	2	2								
ST	MA 2 1.6	MA 2 1.10	MA 2 1.6	MA 2 1.6	MA 2 1.6								
<b>B</b>	Use angle measurement				*select and use <u>benchmarks</u> to estimate measurements of 0-, 45- (acute), 90- (right) greater than 90 (obtuse) degree angles		*identify and justify an angle as acute, obtuse, straight, or right	*use tools to measure angles to the nearest degree and classify the angle as acute, obtuse, right, straight, or reflex	*solve problems of angle measure, including those involving triangles and parallel lines cut by a transversal		solve problems of angle measure, including those involving triangles or other polygons and of parallel lines cut by a transversal		
DOK					2			1	1		1		
ST					MA 2 1.6			MA 2 3.2	MA 2 3.2		MA 2 3.1		
<b>C</b>	Apply geometric measurements			*determine the perimeter of polygons	determine and justify areas of polygons and non-polygonal regions imposed on a rectangular grid	determine volume by finding the total number of the same size units needed to fill a space without gaps or overlaps	solve problems involving the area or perimeter of polygons	solve problems involving circumference and/or area of a circle and surface area/volume of a rectangular or triangular prism, or cylinder			determine the surface area, and volume of geometric figures, including cones, spheres, and cylinders		
DOK				2	3	2	2	2			2		
ST				MA 2 1.10	MA 2 1.10	MA 2 1.10	MA 2 1.10	MA 2 1.10			MA 2 1.10		

Measurement

2. Apply appropriate techniques, tools and formulas to determine measurements -- continued													
	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Algebra I	Geometry	Algebra II	
D									*analyze <u>precision</u> and accuracy in measurement situations and determine number of significant digits	*describe the effects of operations, such as multiplication, division and computing powers and roots on magnitudes of quantities and effects of computation on <u>precision</u> which include the judging of reasonable of numerical computations <u>and their results</u>		apply concepts of successive approximation	
Analyze precision													
DOK									2	2		2	
ST									MA 2 1.7	MA 2 1.7		MA 2 1.6	
F						convert from one unit to another within a system of linear measurement (customary and metric)	*convert from one unit to another within a system of measurement (mass and weight)	*convert from one unit to another within a system of measurement (capacity) and convert square or cubic units within the same system of measurement		*use <u>unit analysis</u> to solve problems	*use <u>unit analysis</u> to solve problems	*use <u>unit analysis</u> to solve problems involving rates, such as speed, density or population density	
Use relationships within a measurement system													
DOK						1	1	1		2	2	2	
ST						MA 2 1.6	MA 2 1.6	MA 2 1.6		MA 4 1.6	MA 4 1.6	MA 4 1.6	

# Data and Probability

1. Formulate questions that can be addressed with data and collect, organize and display relevant data to answer them													
	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Algebra I	Geometry	Algebra II	
<b>A</b>		*pose questions and gather data about themselves and their surroundings	*pose questions and gather data about themselves and their surroundings	*design investigations to address a given question	collect data using observations, surveys and experiments	evaluate data-collection methods	formulate questions, design studies and collect data about a characteristic			formulate questions and collect data about a characteristic which include <u>sample spaces</u> and distributions	formulate and collect data about a characteristic		
Formulate questions													
DOK													
ST		MA 3 1.2	MA 3 1.2	MA 3 1.2	MA 3 1.2	MA 3 1.2	MA 3 1.2			MA 3 1.2	MA 3 1.2		
<b>B</b>	*sort items according to their <u>attributes</u>	*sort and classify items according to their <u>attributes</u>	*sort and classify items according to their <u>attributes</u> and organize data about the items										
Classify and organize data													
DOK													
ST	MA 2 1.8	MA 2 1.8	MA 3 1.8										
<b>C</b>	*create graphs using physical objects	*represent <u>one-to-one</u> correspondence data using pictures and bar graphs	*represent <u>one-to-many</u> correspondence data using pictures and bar graphs	*read and interpret information from <u>line plots</u> and graphs ( <u>bar</u> , <u>line</u> , <u>pictorial</u> )	create tables or graphs to represent <u>categorical</u> and <u>numerical</u> data (including <u>line plots</u> )	*describe methods to collect, organize and represent <u>categorical</u> and <u>numerical</u> data	interpret circle graphs; create and interpret <u>stem-and-leaf plots</u>	select, create and use appropriate graphical representation of data, including circle graphs, <u>histograms</u>	select, create and use appropriate graphical representation of data (including <u>scatter plots</u> ) and <u>box plots</u> ( <u>box and whiskers</u> )	select and use appropriate graphical representation of data and given <u>one-variable quantitative data</u> , display the distribution and describe its shape	select and use appropriate graphical representation of data and given <u>one-variable quantitative data</u> , display the distribution and describe its shape	select and use appropriate graphical representation of data and given <u>one-variable quantitative data</u> , describe its shape and calculate <u>summary statistics</u>	
Represent and interpret data													
DOK													
ST	MA 3 1.8	MA 3 1.8	MA 3 1.8	MA 3 1.10	MA 3 1.8	MA 3 1.2	MA 3 1.8	MA 3 1.8	MA 3 1.8	MA 6 1.8	MA 6 1.8	MA 3 1.8	

Data and Probability

2. Select and use appropriate statistical methods to analyze data													
	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Algebra I	Geometry	Algebra II	
A				*describe the <u>shape of data</u> and analyze it for patterns	*describe important <u>features</u> of the data set	compare related data sets	find the <u>range</u> and <u>measures of center</u> , including <u>median</u> , <u>mode</u> and <u>mean</u>	find, use and interpret <u>measures of center</u> and spread, including ranges	find, use and interpret <u>measures of center</u> , <u>outliers</u> and spread, including range and <u>interquartile range</u>	apply statistical measures of center to solve problems		apply statistical measures of center to solve problems	
Describe and analyze data													
DOK													
ST				MA 3 1.6	MA 3 1.6	MA 3 1.6	MA 3 1.10	MA 3 1.10	MA 3 1.10	MA 3 1.10		MA 3 1.10	
B									compare different representations of the same data and evaluate how well each representation shows important aspects of the data				
Compare data representations													
DOK													
ST									MA 3 1.10				
C										given a scatterplot, determine an equation for a <u>line of best fit</u>		given a scatterplot, determine a type of function which models the data	
Represent data algebraically													
DOK													
ST										MA 3 1.6		MA 3 1.6	

Data and Probability

3. Develop and evaluate inferences and predictions that are based on data													
	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Algebra I	Geometry	Algebra II	
A				*discuss events related to students' experiences as likely or unlikely	*given a set of data, propose and justify conclusions that are based on the data	given a set of data make and justify predictions	use observations about differences between 2 samples to make <u>conjectures</u> about the populations from which the samples were taken	use observations about differences between 2 samples to make <u>conjectures</u> about the populations from which the samples were taken	make <u>conjectures</u> about possible relationships between 2 characteristics of a sample on the basis of scatter plots of the data and approximate lines of fit	make <u>conjectures</u> about possible relationships between 2 characteristics of a sample on the basis of scatter plots of the data			
Develop and evaluate inferences													
DOK													
ST													
B													
Analyze basic statistical techniques													
DOK													
ST													

Data and Probability

4. Understand and apply basic concepts of probability													
	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Algebra I	Geometry	Algebra II	
A						*describe the degree of likelihood of events using such words as certain, equally likely and impossible	use a model (diagrams, list, sample space, or area model) to illustrate the possible outcomes of an event	use models to compute the probability of an event and make conjectures (based on theoretical probability) about the results of experiments				describe the concepts of <u>sample space</u> and <u>probability distribution</u>	
Apply basic concepts of probability													
DOK						2	2	3				2	
ST						MA 3 1.10	MA 3 1.10, 3.2	MA 3 3.8				MA 3 3.1	
B												use and describe the concepts of conditional probability and independent events and how to compute the probability of a <u>compound event</u>	
Use and describe compound events													
DOK												2	
ST												MA 3 3.1	